UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

MODULE	**APM3B10** QUANTUM COMPUTING
CAMPUS	**APK**
EXAM	**NOVEMBER 2014**

DATE: 6/11/2014 **SESSION:** 8:30–11:30

ASSESSOR Prof. W.-H Steeb

EXTERNAL MODERATOR Prof. Y. Hardy

DURATION: 3 HOURS **MARKS:** 40

NUMBER OF PAGES: 3 PAGES

INSTRUCTIONS: ANSWER ALL THE QUESTIONS

ALL CALCULATIONS MUST BE SHOWN

POCKET CALCULATORS ARE PERMITTED

ALL ANGLES ARE MEASURED IN RADIANS

THE PRESCRIBED TEXT BOOKS ARE ALLOWED

QUESTION 1

(a) Consider the Hamilton operator

$$\hat{K} = i\hbar\omega \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

where $\hbar$ and ω (frequency) are constants. Is $\hat{K}$ hermitian? Calculate

$$\exp(-i\hat{K}t/\hbar).$$

Is $\exp(-i\hat{K}t/\hbar)$ unitary? (4)

(b) Consider the initial state

$$|\psi(t=0)\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

Is the state $|\psi(t=0)\rangle$ entangled? Find the state

$$|\psi(t)\rangle = \exp(-i\hat{K}t/\hbar)|\psi(t=0)\rangle$$

and thus solve the Schrödinger equation. (4)

(c) Find the probability

$$|\langle\psi(0)|\psi(t)\rangle|^2.$$

(2)

(10)

QUESTION 2

Consider the normalized state in the Hilbert space $\mathbb{C}^2$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

and the density matrix $\rho(0) = |\psi\rangle\langle\psi|$. Given the Hamilton operator

$$\hat{H} = \hbar\omega\sigma_2,$$

solve the von Neumann equation for the given $\rho(0)$ and $\hat{H}$. The von Neumann equation is given by

$$i\hbar\frac{d\rho}{dt} = [\hat{H}, \rho](t)$$

with the solution

$$\rho(t) = e^{-i\hat{H}t/\hbar}\rho(0)e^{i\hat{H}t/\hbar}.$$

Discuss. **(10)**

QUESTION 3

(a) Consider the 4×4 permutation matrix

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

Find P^2, P^3, P^4. Do the matrices P, P^2, P^3, P^4 form a group under matrix multiplication? Prove or disprove. (5)

(b) Find the eigenvalues and normalized eigenvectors of P, P^2, P^3, P^4. Do all the eigenvalues of P, P^2, P^3, P^4 form a group under multiplication? Prove or disprove. (5)

(10)

QUESTION 4

Consider the vector in the Hilbert space $\mathbb{C}^6$

$$\mathbf{v} = \begin{pmatrix} i \\ -i \\ 0 \\ 0 \\ i \\ -i \end{pmatrix}.$$

(a) Can the vector $\mathbf{v}$ be written as

$$\mathbf{v} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} ?$$

(4)

(b) Can the vector $\mathbf{v}$ be written as

$$\mathbf{v} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \otimes \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} ?$$

(4)

(c) Normalize the vector $\mathbf{v}$ and then find the corresponding density matrix. (2)

(10)

END OF QUESTION PAPER